ON SOURCES OF STELLAR ENERGY—A CRITICISM OF THE BETHE-GAMOW THEORY*

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ABSTRACT. A review of the existing theories of energy production in white dwarfs and other stars has been made and it has been shown that the existence of neutrons and high pressure due to the degenerate electron gas inside the white dwarfs may explain the low energy production in the white dwarfs.

In a series of papers Weizsäcker (1937), Bethe and Critchfield (1938), Bethe (1939), Gamow (1939), Gamow and Teller (1939) have applied recent researches in nuclear physics to the explanation of the production of energy in stars and to stellar evolution. By applying the formula of Atkinson and Houtermans (1929), as improved by Gamow and Teller (1938) for the probability of a nuclear reaction in a gas obeying the Maxwellian distribution of velocities, Bethe (1939) has shown that the energy production in the stars of the main sequence is due to carbon and nitrogen acting as catalysts, the net result being the formation of a helium nucleus and two positrons out of the four protons. Part of the surplus energy is radiated away in the form of two neutrinos when N^{13} and O^{15} disintegrate into C^{13} and a positron and into N^{15} and a positron respectively. In each cycle therefore we get 4.0×10^{-5} ergs of useful energy, part of which is radiated away in space and part is used in raising the temperature of the star.

As a result of this cycle of reactions, the star decreases a little in mass but grows brighter, hotter, and a little larger, till, according to Gamow (1939), the energy production due to the cycle is no longer able to maintain the temperature of the star and a contractive evolution sets in, which is very rapid if the matter inside the star continues to obey the laws of an ideal gas. Before long, however, degeneracy sets in which checks this rapid contraction and the star reaches the white dwarf stage.

For the evolution of the red giants, Gamow and Teller (1939) have shown that energy production by the carbon-nitrogen cycle and due to the direct combination of protons is negligible, and that in this case the production of energy is due to the reaction of protons with the lighter elements H^2, H^3, Li^6, Li^7, Be^9, Be^{10}

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and B^{11}. They have shown that definite bands exist in the radius-luminosity diagram which correspond to energy production by different elements. It is the object of this paper to point out some of the difficulties in the evolution of the red giants and the white dwarfs and an attempt has been made to explain them.

WHITE DWARFS

The behaviour of the white dwarfs is very peculiar and puzzling in as much as their production of energy is extremely low for their accepted values of temperature and density. As has been emphasized by Gamow (1939), we should have to assume very low temperatures ($\sim 10^6$ degrees) for the interior of white dwarfs in order to bring down the energy production due to the reaction $H^1 + H^1 \rightarrow H^2 + \beta^+$ to the observed values.* On the other hand, if we preclude the presence of hydrogen, the energy production by any other reaction will be negligible even for white dwarfs up to very high temperatures. Table I shows that the energy production by $C^{12} + He^4 = O^{16}$ reaction is very small even up to temperatures of the order of eighty million degrees and for a density of the order of 10^6. At lower temperatures the energy production is negligible. Calculations have been made from the formula given by Bethe (1939), p. 434, formula (16).

TABLE I

Temperature in million degrees	τ	$p/\rho x_1$	p for $\rho x_1 = 10^6$.	Energy in ergs gm^{-1} sec^{-1}.
20	119	3.6×10^{-21}	3.6×10^{-15}	4.2×10^{-20}
40	94.5	1.04×10^{-10}	1.04×10^{-4}	1.2×10^{-9}
60	82.5	1.23×10^{-5}	1.23×10	1.4×10^{-5}
80	75.0	1.93×10^{-2}	1.93×10^{4}	2.2×10^{-1}
100	69.6	3.55	3.55×10^{6}	4.1×10

But values of the mean molecular weight from different theories of internal structures of stars show that some of the white dwarfs still contain nuclei of atomic weight one. For example, according to Chandrasekhar (1939), Sirius B contains about 52% hydrogen and Van Mannen star No. 2, 66%. We cannot therefore assume a low temperature for the interior of the white dwarfs. Calculations of the temperature variation in the outer atmosphere, where ordinary gas laws are expected to hold, show that the temperature reaches a value of the order

* At low temperatures, the energy production by the carbon cycle is negligible in comparison with the energy production due to the above reaction. See Bethe (1939), Fig. 1, d 452.

of 20×10^6 degrees before degeneracy sets in [see Kothari (1933) and Strömgren (1937)]. The temperature in the interior must, if anything, be higher than this value.

The only way out of this difficulty seems to be to suppose that in the interior of the white dwarfs either (a) there are certain processes which liberate a very small amount of energy, or (b) there are some endothermic reactions going on which partly counterbalance the energy production and thus cause the net production of energy to be appreciably reduced. In the last case we have to give up partly the non-equilibrium theory of energy production which is the basis of Bethe and Gamow's works.

THE NEUTRON HYPOTHESIS

Since irrespective of the presence of other nuclei, the hydrogen nuclei themselves cause the evolution of more energy in the white dwarfs than is actually observed, the supposition has been made that hydrogen nuclei are totally absent from the interior of white dwarfs and the reduction in the observed mean molecular weight of the white dwarfs is due to the presence of neutrons. The formation of neutrons at very high densities and not too high temperatures was first indicated by Sterne (1933). The pressure due to the degenerate electrons being much larger than that due to other nuclei, their disappearance will ultimately lead to a reduction of pressure. But such a process will not lead to the liberation of vast amount of energy due to gravitational contraction, as has been supposed by Baade and Zwicky* (1924), as the neutronic mass is greater than the combined mass of a proton and an electron.

We cannot, however, suppose that all nuclei inside the white dwarfs have been transformed into neutrons. The effective opacity co-efficient of white dwarfs requires that other nuclei should also be present in their interior.

Since Bethe (1939) has shown that during the earlier history of a star all elements lighter than carbon, except He^4, are converted *into* He^4 *and that the abundance of* C^{12} *and* O^{16} remains practically unchanged, inside the white dwarfs, we must have neutrons, He^4, C^{12} and O^{16} in addition to other heavy nuclei. In view of the fact that fission is produced by neutrons in heavier nuclei, we shall, however, exclude the discussion of heavier nuclei and confine our attention only to the above four.

We have already shown that up to temperatures of the order of 50 million degrees, which may be taken to be a probable value of temperature for the interior of the white dwarfs, there will be no mutual interpenetration and

* The total gravitational energy released if the Sun contracts to one-thousandth of the present radius will be of the order of 10^{51} ergs. This will be just sufficient to convert all atomic nuclei into neutrons.

occurrence of reaction possible between the heavier nuclei. We shall therefore only consider the action of neutrons on the other three nuclei.

He^4

Although we have got some experimental evidence on the scattering of neutrons by helium, we have no experimental evidence either of capture of, or disintegration by, neutrons in the case of helium. We have therefore to be guided solely by energy and probability considerations. The following show all the possible reactions between He^4 and neutrons :

$$He^4 + n^1 \longrightarrow He^3 + n^1 + n^1 \qquad \ldots \quad (1)$$

$$He^4 + n^1 \longrightarrow H^4 + H^1 \qquad \ldots \quad (2)$$

$$He^4 + n^1 \longrightarrow H^3 + H^2 \qquad \ldots \quad (3)$$

$$He^4 + n^1 \longrightarrow He^5 + \gamma \qquad \ldots \quad (4)$$

Reactions of the type (1) and (2) are well known, but reactions of type (3) have not been observed. But in view of the fact that Chadwick and Goldhaber (1935) have observed the emission of H^3 by bombarding Li^6 and B^{10} by slow neutrons (the reaction according to them has a cross-section of 10^{-21} cm.^{2}) and Oliphant, Kempton and Rutherford (1935) have observed the emission of H^2 by bombarding Be^9 by protons, (3) may be possible with some of the lighter nuclei.

He^5 has been observed in certain disintegration experiments, but there is some doubt regarding its stability. Most probably it is just stable. (See a letter by Sirkar and Rai in 'Science and Culture', February, 1940.)

Of the above reactions the first three are endothermic and the energy of the bombarding neutron, from mass considerations, comes out to be so large for the reactions to be possible that there will be a negligible number of neutrons with those energies in the interior of the white dwarfs. After making even the most favourable assumptions for the probability of the reactions, it can be shown that they will be of little importance. We shall therefore consider only the reaction (4) in the case of He^4.

C^{12}

Unlike helium, a large number of reactions have actually been observed with carbon. These are summerised below :

$$C^{12} + n^1 \qquad Be^9 + He^4 \qquad (5)$$

$$C^{12} + n^1 \qquad 3He^4 + n^1 \qquad (6)$$

$$C^{12} + n^1 \qquad B^{12} + n^1 \qquad (7)$$

$$C^{12} + n^1 \qquad C^{11} + n^1 + n^1 \qquad (8)$$

$$C^{12} + n^1 \qquad C^{13} \qquad (9)$$

With the exception of the last reaction, all other reactions listed above are endothermic requiring large energies (order of several Mev.) for their realisation. The reaction (9) listed above will therefore be the only reaction which will be of importance. Similar considerations with oxygen show that radiative capture will be the only process which needs to be considered.

Our knowledge of the cross-section for thermal neutrons in helium, carbon and oxygen is at present very scanty. Unless however low energy resonance levels are present in these elements, the cross-sections must be very small and must decrease as the energy of the neutron is increased. Carrol and Dunning (1938) have obtained a value 1.51×10^{-21} for the scattering cross-section of thermal neutrons in helium, and according to Straub and Stephens (1939), who have measured the ratio of cross-section of neutrons in helium and hydrogen, no appreciable increase in the ratio occurs till the neutrons reach an energy of 0·8 Mev. As this is too high for the thermal neutrons in the stellar interiors, we shall not consider the modification introduced in the value of the cross-section by the presence of resonance levels.

A similar estimate of the nuclear cross-section for thermal neutrons in the case of carbon and oxygen is not possible. According to Amaldi, Bocciarelli, Rasetti and Trabacchi (1939) and Goloborodko and Leipunski (1939), the cross-section for neutrons of energies 130 Kev. to 350 Kev. is of the order of 2×10^{-21} cm.^{2} both for carbon and oxygen.

We can therefore put $\sigma \simeq \dfrac{\pi h^2}{2mE}$.

Putting this value and integrating, we shall get the total number of collisions. This is to be multiplied by the ratio of the probabilities of γ-ray and particle emissions which is

$$\frac{\Gamma r}{h/mr^2} = \frac{\Gamma mr^3}{h^2}$$

where Γ is the γ-ray width in ergs and r the nuclear radius.

Table II gives the γ-ray width for the different reactions, calculated according to Bethe (1939) (eq. 10).

TABLE II

Original nucleus.	Product nucleus.	γ-ray energy in MMU.	γ-ray width in ergs.
He^4	He^5	~ 0.02	0.64×10^{-14}
C^{12}	C^{13}	5.69	4.50×10^{-12}
O^{16}	O^{17}	4.75	3.14×10^{-12}

The total number of captures per sec. per gram is finally given by

$$N = \int_0^\infty \frac{4xy\rho\,\sigma}{m_1 m_2 (kT)^{\frac{3}{2}}} \left(\frac{2\pi}{m}\right)^{\frac{1}{2}} \frac{\Gamma m_1^2}{h^2} e^{-E/kT} E\,dE$$

$$= \frac{\pi x y \rho}{m_1 m_2} \left(\frac{2\pi}{m\,kT}\right)^{\frac{1}{2}} \Gamma_1^2$$

$$\simeq xy \frac{(A_1 + A_2)^{\frac{7}{2}}}{(A_1 A_2)^{\frac{3}{2}}} \Gamma \times 6.58 \times 10^{44}$$

where A_1, A_2 = Atomic weights of the nuclei and the neutron

$$m = \text{reduced mass} = \frac{m_1 m_2}{m_1 + m_2}.$$

This shows that even for helium for which the probability of capture is the least, the number of captures gm.$^{-1}$ sec.$^{-1}$ will be of the order of 10^{24} This will lead to a very large amount of energy being produced in the white dwarfs.

It thus appears that the reverse processes will play a large part in these reactions. Taking for example C^{12} as a representative case, C^{13} will either be transformed into C^{14} or C^{12} by the neutron bombardment. The latter process will absorb energy. Also on account of the degeneracy of the electrons, the transformation of C^{14} into N^{14}, an electron and a neutrino will not be possible as this will lead to an increase of pressure due to the degenerate electron gas. These will therefore be ultimately broken up by neutrons and building of heavier nuclei will be a very very slow process. It may thus happen that ultimately very little production of energy takes place.

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